

## Comment on DIRAC FORMULATION OF FREE OPEN STRING

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Quantization of constrained physical systems is one among the most important and, simultaneously, the most difficult problems of contemporary theoretical physics. In fact field theorists badly need to know how to quantize gauge field models, string theories and Einsteinian gravity which, according to our best knowledge, all are examples of field theories with constraints for which simple standard methods of fields quantization do not work. Although many scientists are working on the problem of quantization of constrained systems and although to find its universal solution is really urgent and necessary for further development of quantum field theory the only self-consistent and manageable approach to the problem is the method of quantization invented more than 40 years ago by P.A.M Dirac. Despite of its successes and mathematical beauty the Dirac method leaves many questions open and in order to understand its properties, as well as its physical implications, it is useful to analyse carefully some particularly chosen models. Burdik and Navratil do

this by investigating the free open string in dimension  $D$ . They develop step by step classical canonical formalism for such a model and arrive at sets of Dirac brackets for two choices of gauge conditions: the so-called light cone gauge being Lorentz noncovariant and Lorentz covariant gauge generalizing the condition proposed by Rohrlich about thirty years ago. Obviously, to get classical Dirac brackets is only a primary step if one wants to investigate a quantum system. The real challenge is to find a representation of an operator algebra obtained by formal replacement of Dirac brackets by commutators. Trying to solve this problem Burdik and Navratil are partially successful - for the light cone gauge they find a Fock-type representation which for  $D = 2$  and  $D = 26$  is consistent with results following from standard classical theory. Unfortunately, they do not give an explanation whether these specific values of  $D$  have any physical meaning or are purely incidental. Investigating the Lorentz covariant gauge the authors arrive at results which for me seem more intriguing. For this case the quantization of Dirac brackets leads to the algebra in which canonical commutation rules between coordinate  $Q^\mu$  and momentum  $P^\nu$  operators are modified by terms proportional to  $P^\mu P^\nu$ , momenta commute and commutators between coordinates are proportional to the components of angular momentum. Such an algebra coincides with noncanonical algebras found in investigations rooted in quantum gravity and leading to theories with fundamental length [1] and for the so-called Wigner quantum systems [2], [3]. More detailed investigation of such a coincidence seems to be an interesting problem as well as it would be extremely useful to find representations of this algebra. This, I believe, will be the subject of Burdik and Navratil's further research.

## References

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